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Dead time corrections using the backward extrapolation method

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ABSTRACT

Dead time losses in neutron detection, caused by both the detector and the electronics dead time, is a highly nonlinear effect, known to create high biasing in physical experiments as the power grows over a certain threshold, up to total saturation of the detector system. Analytic modeling of the dead time losses is a highly complicated task due to the different nature of the dead time in the different components of the monitoring system (e.g., paralyzing vs. non paralyzing), and the stochastic nature of the fission chains. In the present study, a new technique is introduced for dead time corrections on the sampled Count Per Second (CPS), based on backward extrapolation of the losses, created by increasingly growing artificially imposed dead time on the data, back to zero. The method has been implemented on actual neutron noise measurements carried out in the MINERVE zero power reactor, demonstrating high accuracy (of 1–2%) in restoring the corrected count rate.

1. Introduction

Dead time effect in neutron detections, caused by both the detector and the electronics dead time, is a highly nonlinear effect, known to create high biasing in physical experiments as the power, and hence the count rate, grows over a certain threshold [1,2]. For sufficiently high power, the system might be totally saturated, but even in low power levels (as demonstrated in this paper), loses might reach up to 30%.

Analyzing neutron detector readings is perhaps one of the most basic aspects in nuclear engineering. The detector count rate is a basic observable of a nuclear core, and is used both when operating the reactor (approach to criticality experiments, the regulation system, the SCRAM system, etc.) and when conducting in-pile experiments. Therefore, quantification of the dead time losses is of utmost importance in reactor monitoring and in-pile experiments.

Mathematical modeling the dead time losses is a highly complicated task due to different nature of the dead time in the various components of the monitoring system (e.g., paralyzing vs. non paralyzing), as well as the stochastic nature of the fission chains and the variance in the dead time itself (which might not be constant). Although analytic treatment of the dead time effect was largely studied, from early works [3,4] up to very recent studies [5,6], a full analytic treatment is still not available. Therefore, most applicable models depend on phenomenological models, where the empirical data is typically fitted on exponential models [7–9].

In the present study, a new method is introduced, referred to as the Backward EXtrapolation method (BEX), to evaluate the dead time correction of neutron Counts Per Second (CPS) rate in nuclear reactors (and specifically, in the context of reactor monitoring). The method was originally suggested for dead time correction in Neutron Multiplicity Counting (NMC) [10], but to the best of our knowledge, was never implemented in reactor monitoring. While the theoretical background for the method in both NMC and reactor monitoring is very much the same, the two are characterized by (typically) very different count rates: the count rate in reactor monitoring is often several orders of magnitude higher than the expected count rate in NMC.

The method described in the present study has one very interesting and unique feature that distinguishes it from most existing methods: it does not assume a-priory any knowledge on the functional dependence between the duration of the dead time and the count loss (or, equivalently, the correction).

The performance of the method are demonstrated and evaluated by implementing it to a set of seven measurements, corresponding to seven different power levels, performed in the MINERVE zero power reactor [11,12] during June 2015. A very good correspondence exists between the results obtained using the BEX method and the empirical results.

Although, as argued later, the method preforms respectively good in the experiments analyzed, the data set is fairly restricted; the power levels are relatively low (0.2–80 W), count rates are between 5.50×10^4 to 1.64×10^6 cps, the maximal estimated dead time losses are bounded by 31%, and all measurements were taken on the same critical

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configuration. Therefore, although the proposed scheme is referred to as a "method", it should be clear that the purpose of this study, at this point, is to form a proof of feasibility for the method, and further validation should be conducted before the method can be claimed operational.

The paper is arranged in the following manner: Some relevant theoretical considerations on dead time losses are introduced in Section 2. The BEX method is described in full details in Section 3. Sections 4 and 5 describe the experimental results obtained using the method. Section 6 holds a discussion on the uncertainty analysis of the introduced method and the conclusions are given in Section 7.

2. Theory of different dead time models

2.1. Dead time definition

The term *dead time*, in the present context, comes to describe a time period after a detection, in which the acquisition system is not operational. By acquisition system we refer to both the physical and the electronic components. In other word, a neutron arriving at the detector during the dead time will not be recorded. In the literature, two distinctions are considered regarding the nature of the dead time: constant vs. varying and paralyzing vs. non paralyzing [5]. In the first distinction, one separates between a dead time of a fixed duration and a dead time whose duration is a random variable. The second distinction regards the following question: if a neutron arrives to the detector while the detection system is down due to a dead time inflicted by a previous detection, will it once again inflict a dead time, extending the duration in which the system is down?

To demonstrate this point, we refer the reader to Fig. 1. Each point indicates an arrival time of a neutron to a detection system characterized by a dead time τ. Clearly, the second neutron arriving (indicated by the green dot) will not be recorded, but what about the third neutron (indicated by the black dot)? In the non paralyzing model, the fact that the second neutron arrival was "shielded" implies that as far as the acquisition system is concerned, it never existed, and thus the third detection (indicated by the black dot) ***is recorded***. In the paralyzing model, the second arrival - recorded or not - will inflict a dead time τ and the third detection ***is not recorded***. The term "paralyzing" for describing the second model reflects the fact that in a paralyzing setting, once a certain threshold is met, further increase in the reactor power results in a decrease of the detection rate, up until the acquisition system is totally saturated with no recordings at all.

From a physical point of view, the nature of the dead time is determined by the component creating the dead time. Typically, dead time due to the electronic registration system is considered to be non paralyzing, while dead time created by the physical process in the detector is often paralyzing. Since the detection mechanism must have both, the true nature of the dead time is not strictly paralyzing or non paralyzing, but actually a combination of the two.

2.2. Why is it so difficult do model the dead time?

Most of the well known models for reactor kinetics, from the time dependent Boltzmann transport equation to the point reactor kinetic equations, are deterministic. As such, they consider the neutron population size or the detection rate in a time interval as a deterministic function of time, whereas physically both are random variable. The stochastic nature of the detection signal originates from both the probabilistic properties of nuclear interactions and the statistical nature of the detection process.

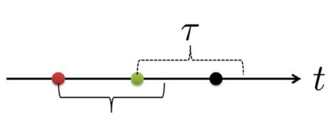

Fig. 1. Paralyzing vs. non paralyzing dead time. In the non paralyzing model, the third detection (indicated by the black dot) is recorded, whereas in the paralyzing model it is not.

In a more formal approach, consider a detection signal

$$D_T = (t_1,\ t_2, \ldots, \ldots, t_i, \ldots, \ldots, t_n), \qquad t_i < t_{i+1}, \tag{1}$$

describing the detection times in a nuclear system during an interval of duration T. D_T is, in fact, a single *realization* of a random process, which depends on many parameters characterizing the nuclear system, among them is the dead time τ. Since the present study focuses on the dead time and the detection rate (counts per second - CPS), the random process is denoted by $X(T,\ \tau)$.

The measured CPS, given by n/T, is a *sampling* of the average count rate in the random process $X(T,\ \tau)$. The average losses due to a dead time τ, which are defined by some functional form $f(\tau)$, depend of course on the average CPS. Moreover, the average losses are also strongly dependent on temporal correlations between the detections. For instance, in a marginally subcritical system exposed to a weak neutron source, detections typically tend to be concentrated in dense bursts [13] and the dead time effect is enhanced compared to a system characterized by random uncorrelated detections. Consequently, an analytic model for the dead time effect must be a stochastic model.

Typically, most stochastic models of the count distribution (neglecting dead time effect) are based on the construction of a differential model for the so called probability generating function [14]. These models were previously extended to incorporate the dead time effect [6,15] but in an extremely limited setting, since differential models, whose solutions are uniquely determined by the initial conditions, can only model Markovian processes, whereas the effect of the dead time is not Markovian.

The dead time is a non Markovian effect because in the context of differential models, the state of the system at time t is defined by the size of the neutron population at time t and the number of detections at time t. However, in order to account for dead time effect, the model must "remember" when the previous detection was recorded. In fact, in a non paralyzing setting, the model must have recollection of ***all*** arrival times up until t. Accounting for such "memory" effects is typically done by using integral models. This approach was used in [5], where the so called integral master equation was derived. However, the high complexity of the integral model dramatically limits the results and raises extreme difficulties in obtaining any explicit formula for the count rate distribution.

Nonetheless, although explicit analytic results obtained so far do not have any real implementation in full reactor settings (all the studies referenced above were done in the setting of neutron multiplicity counting, and, to the best of our knowledge, were never implemented in nuclear reactor cores, experimental or operational), they do provide a piece of very important information: the functional form of the average count losses, i.e., $f(\tau)$, is an analytic function with respect to τ. From a theoretical point of view, this is the only assumption used in the BEX method.

3. The backward extrapolation method

3.1. Description of the BEX method

The basic idea in the Backward EXtrapolation (BEX) method is to construct the function describing the reduction in CPS due to the dead time effect, by artificially imposing a growing dead time on the signal, and then extrapolate the resulting function back to zero.

As before, the CPS in a system with a dead time τ is denoted by $f(\tau)$, the actual measured CPS by CPS_0, and the actual dead time (which might be unknown) by τ_0.

Looking at the detection signal D_T measured (or observed) on an interval of duration T (Eq. (1)), the measured sample average CPS,

$$CPS_0 \equiv CPS(\tau_0) = \frac{n}{T}, \tag{2}$$

is a sampling of $f(\tau_0)$. Notice that for every dead time $\tau \geq \tau_0$, $f(\tau)$ can be explicitly sampled from the measured data D_T by eliminating all detections t_j such that $t_j - t_{j-1} < \tau$. In a non paralyzing dead time the procedure is a bit more complicated but still rather straightforward. For instance, one option for imposing non paralyzing dead time is to successively progress from one detection to the next, and at each point decide if the detection is shielded, by considering both the duration between the detections, and whether the first detection of the two was recorded.

These procedures of artificially imposing dead time on the measured data may be repeated for a series of increasing values of $\tau \geq \tau_0$. Clearly, for every $\tau \leq \tau_0$, $f(\tau)$ can not be evaluated by artificially imposing dead time, since imposing a dead time shorter that the actual dead time would not have any effect on the CPS. At this point, the resulting sampled $f(\tau)$ can be fitted with any chosen model on all sampled points of $f(\tau)$ (for $\tau \geq \tau_0$) and extrapolate the fit back to zero, i.e., $\tau \to 0$. This procedure enable the evaluation of the corrected CPS, i.e., CPS($\tau \to 0$), given by

$$f(0) = \lim_{\tau \to \infty} f(\tau) \tag{3}$$

Fig. 2 describes the full implementation of the method on an actual measurement taken at the MIN- ERVE reactor at power level of 80(± 3%) W, producing 1.6461×10^6 CPS. The x-axis describes the value of the induced dead time τ and the y-axis the measured CPS. A cutoff at $\tau = \tau_0 = 150 \pm 10$ ns is clearly observed, indicating this is the dead time of the system during the signal measurements. The blue dots represent the resulting CPS after artificially imposing dead time on the measured signal and the red curve represents a fourth degree polynomial fitted on the data for $\tau > \tau_0$ (since all points are, in fact, the same measurement, a non weighted fit was executed). The red curve is an actual realization of the theoretical CPS function $f(\tau)$. Thus, the corrected CPS is obtained by extrapolating $f(\tau)$ backwards to $\tau \to 0$ and is given by the bisection between $f(\tau)$ and the y-axis (purple dot in Fig. 2). The choice of a very general fit model, i.e., a fourth degree polynomial, is arbitrary, manifesting two facts mentioned earlier in the paper: first, the assumption that $f(\tau)$ is analytic, and second, that no prior knowledge on $f(\tau)$ is needed.

3.2. Implementation of the BEX method with different dead time models

The procedure described above can be implemented using any model for the dead time. However, when implementing the method with a non paralyzing dead dime, a certain deviation is expected, due to the fact that under certain circumstances, some unrecorded detection

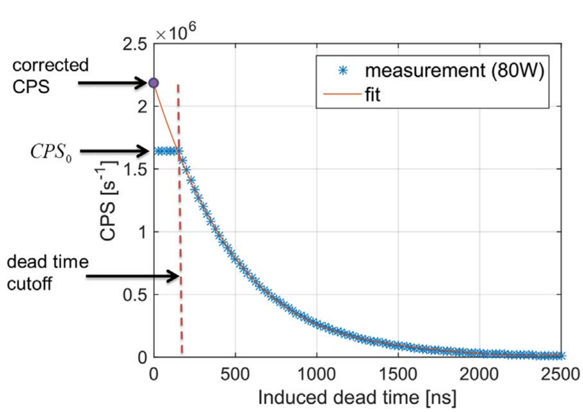

Fig. 2. Full implementation of the BEX method on actual neutron detection signal measured at the MINERVE reactor at power level of 80 W. The dead time of the system during the measurement is clearly observed at τ=150 ns.

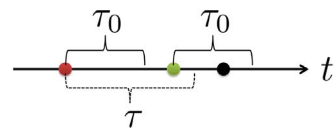

Fig. 3. Artificially imposing non paralyzing dead time $\tau > \tau_0$ on the detection signal is expected to result in certain deviation due to the need to "revive" unrecorded detections. See text for more details.

events should be "revived". To demonstrate this point, the reader is referred to Fig. 3.

Consider three neutrons arriving consecutively to the detector at times t_1 (red dot), t_2 (green dot), and t_3 (black dot), in a system with real dead time τ_0. According to the definition of dead time, the detection at t_3 is not recorded. Now, assume a non paralyzing dead time $\tau > \tau_0$ is artificially imposed on the data, as shown in Fig. 3. Now, the arrival at t_2 should be deleted, but since a non paralyzing model use assumed, the arrival at t_3 should be "revived", which is impossible since it was never recorded.

On the other hand, this is a "local" effect in the sense that once the imposed dead time is large enough, it will cover the detection in t_3 anyway, and the imposed dead time will once again be valid (with respect to the three detections in Fig. 3). Generally, the larger the imposed dead time with respect to the actual one, the smaller the expected deviation due to un-revivable detections is.

The implantation of the BEX method on the exact same measurement used in Fig. 2 with both paralyzing and non paralyzing dead time is shown in Fig. 4. The corrected CPS are 2.1727×10^6 s^{-1} and 2.0579 $\times 10^6$ s^{-1}, respectively, showing a 5% discrepancy in the corrected CPS. As stated earlier, implementation of the non paralyzing model is bound to give a bias in the corrected CPS, but quantifying it is beyond the scope of this paper. Therefore, it should be clear that implementation of the BEX method in a non paralyzing dead time setting, at this point, is questionable.

4. Experimental results

4.1. Experimental setting

In order to validate the BEX method, a set of seven measurements was performed in the MINERVE reactor, differing only in the power level of the reactor. The MINERVE reactor, a part of the CEA Cadarache Center, is a light water moderated zero power reactor (ZPR), dedicated mainly to nuclear data experimental validation using

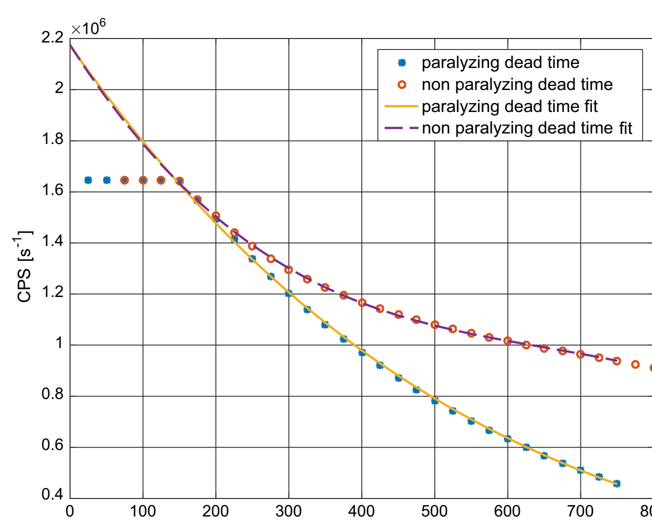

Fig. 4. Implementation of the BEX method on a detection signal from the MINERVE reactor, at 80 W, using both paralyzing and non paralyzing dead time models.

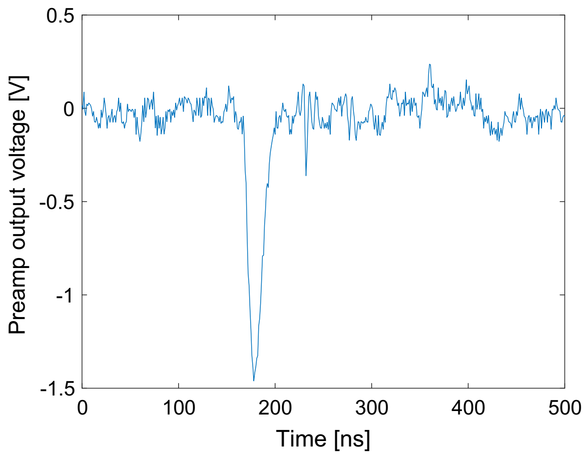

Fig. 5. Pre-amplifier response to a single neutron detection.

a variety of experimental techniques [11].

The experiment was performed in the framework of a tripartite collaboration between CEA, Ben-Gurion University of the Negev (BGU) and the Israel Atomic Energy Commission (IAEC) utilizing the core configu- ration of the MAESTRO-SL experimental program. The experiment detailed here was performed alongside another, different, oscillation measurement campaign using high sensitivity CEA made miniature fission chambers [16] located at the center of the core. The detector used was FC n◦ 2295, 8 mm in diameter with ~10 mg of ^{235}U. The detector was connected to a Canberra ADS7820 amplifiers/discriminator. This analog module converts input current pulses to voltage and then applies a pulse discrimination based on a variable threshold (typically 0.5 V). The output of the module is a TTL pulse train (width approximately 50 ns, see Fig. 5). Signal acquisition was made using a Fast-Comptec MCA-3 acquisition card. Fig. 5 below shows the preamp output for a single neutron detection. An additional width to the dead time is due to the amplifier/discriminator (estimated by 50 ns) and the electronic dead time from the Fast-Comptec acquisition card.

The measurements took place on June 3rd, 2015, in seven different power levels: 0.2 W, 1 W, 5 W, 10 W, 20 W, 50 W and 80 W. Power levels were measured using a calibrated fission chamber. Uncertainty on the absolute power level is 3% and the relative uncertainty (between the reported values in the present experiment) is 0.9%. The reactor was critical during the measurements at each power level, each measurement was about 15 min long, and no significant power drift was recorded. Table 1 below shows the total CPS measured in each experiment.

Generally, the neutron count rate is expected to be linear in the power. The zero power count rate was not measured, but when fitting the first two measurements - which are piratically zero power, and the expected dead time losses are very small - and extrapolating to zero power, we result with 27 cps, practically zero. There for, it is safe to assume that the zero power detector readings are negligible and do not create any biasing in the results. Looking at Table 1, the count rate exhibit a clear sublinear behavior as a function of the power (plotted on Fig. 7), indicating a major drop in the count rate due to detector dead time. For example, the count rate in the last measurement at 80 W is below the linear fit by 33%.

Table 1
Total neutron count rate vs. reactor power.

power [W]	CPS [$\times 10^6$ s^{-1}]
0.2	0.0055 ± 1.42%
1	0.0275 ± 0.60%
5	0.1323 ± 0.28%
10	0.2618 ± 0.19%
20	0.4989 ± 0.14%
50	1.1410 ± 0.09%
80	1.6461 ± 0.08%

Since the count rate in the first measurement (0.2 W) is considerably low, no significant dead time losses are assumed in this measurement. Consequently, the linear curve connecting the origin with this point (0.2 W, 5.5×10^3 s^{-1}) is used as the "zero dead time" reference CPS value to estimate the performance of the method.

4.2. Full implementation of the BEX method

The BEX method was implemented on each of the seven measurements, using the paralyzing dead time model. The imposed artificial dead times range from τ=25 ns up to τ=2500 ns with 25 ns increments and the fit was done using a fourth degree polynomial function. From a practical point of view, this means that no specific model assumptions were made and the most general model was used. The results, both the sampled values of $f(\tau)$ and the fit, for all seven measurements are shown in Fig. 6. In all measurements, the smallest artificially imposed dead time for which a drop in the count rate occurs is found to be τ=150 ns, indicating the actual dead time τ_0.

As expected, the effect of the artificially imposed dead time on the count rate increases as the power increases, starting from 1.2% loss for τ=2500 ns at 0.2 W and up to approximately completely paralyzed system for the same value of τ at 80 W. In all the graphs shown in Fig. 6 the bisection of the fitted curve (red line) on the CPS axis corresponds to the corrected value of the CPS. The results are summarized in Table 2 and Fig. 7.

The goodness of fit was evaluated using $\chi^2 = \frac{1}{n}\sum_{i=1}^{n}[Y_i - Y(\tau_i)]^2$, where n is the number of points used for fit, $Y(\tau_i)$ is the measured CPS value (after artificially imposing dead time τ_i), and Y_i is the fit value at τ_i. The maximal value of χ was 0.3% (with respect to the measured CPS) and all other values were considerably lower, i.e., less then 0.01% (see Section 6).

The corrected CPS values exhibit an good agreement with the reference CPS values, with a maximal error of 1.8% and an average error of 0.7%. The uncertainty on the reference value is taken as the relative uncertainty on between the power levels. The uncertainty on the corrected CPS will be described in more details is Section 6.

4.3. Implementation of the BEX Method using Exponential fit

The results obtained so far show very good agreement with the reference CPS. However, two notes are worth mentioning regarding the fact that the data was fitted and then extrapolated back to zero using a fourth degree polynomial. First, as a general note, a fourth degree polynomial generally suffices to fit any monotonic function. Thus, no physical insights into the nature of the dead time losses can be inferred form the fit results (i.e., the polynomial coefficients). Second, the fact that $f(\tau)$ has a finite limit as τ tends to infinity, whereas a fourth degree polynomial does not, requires that a large number of induced dead times is used.

In order to account for both issues, the implementation of the BEX method was repeated, replacing the fourth degree polynomial with a simple exponential of the form $f(\tau) = Ae^{-\Lambda\tau}$. Since the exponential model implies that $f(\tau)$ vanishes as τ tends to infinity, this model is only suitable for a paralyzing dead time model. It is also worth mentioning that this notation implies that the corrected CPS is nothing more than the multiplicative factor A. The results are shown in Table 3 below.

According to Table 3, for total count rate below 0.5×10^6 s^{-1} (corresponding to 20 W), the difference between the two models is negligible. As the count rate increases, we see some bias, but the results are still very similar, with a maximal 4.9% discrepancy between the two

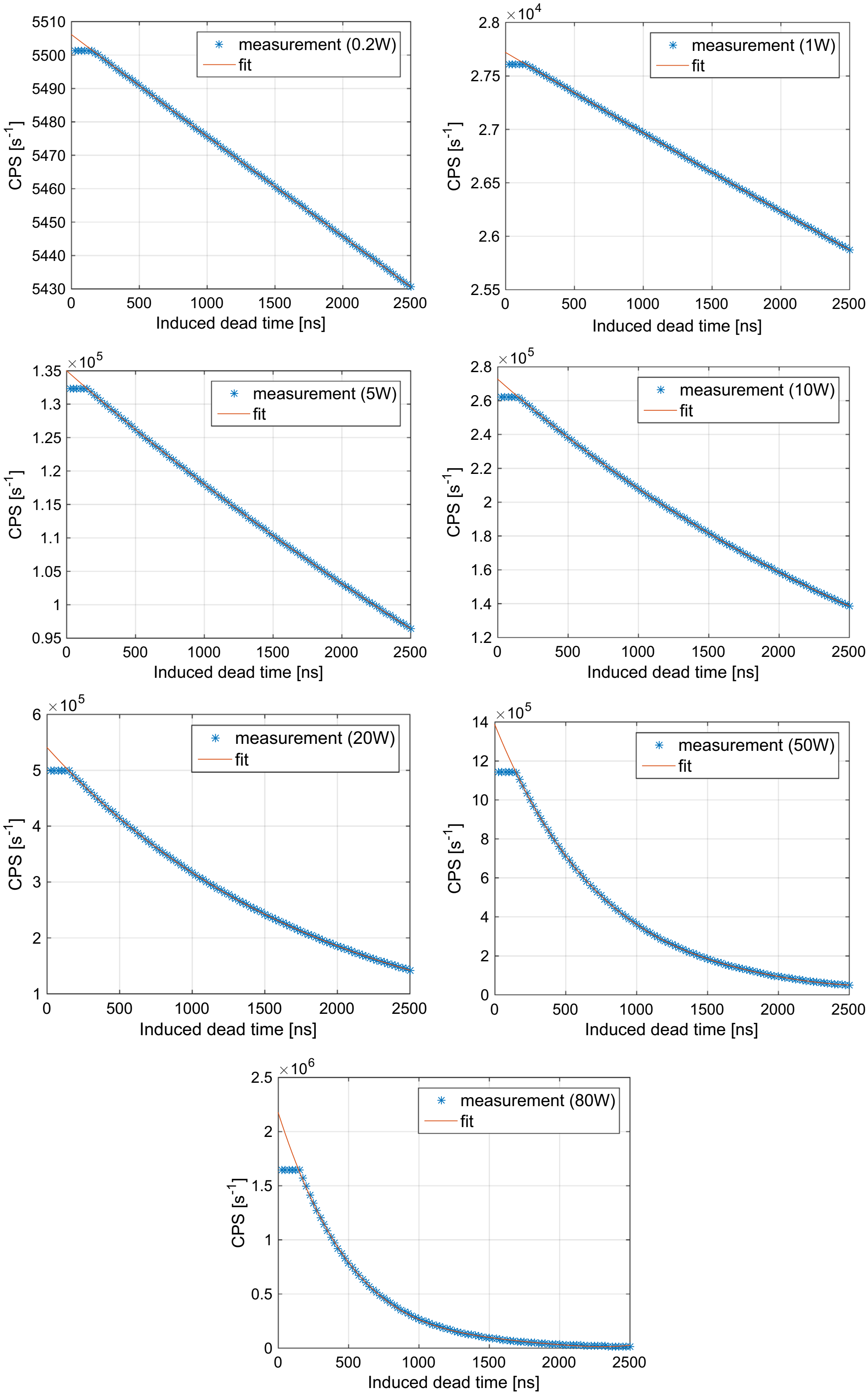

Fig. 6. Implementation of the BEX method (with paralyzing dead).

Table 2
Dead time corrections using the BEX method and the reference CPS (linear fit). All CPS values are given in units of [10^6 s^{-1}].

Power [W]	CPS_0	corrected CPS BEX,4th order	correction [%] (BEX/CPS_0 − 1)×100	reference CPS
0.2	0.0055 ± 1.4%	0.0055 ± 0.0%	0.1	–
1	0.0275 ± 0.6%	0.0277 ± 0.1%	0.4	0.0275 ± 0.9%
5	0.1323 ± 0.3%	0.1350 ± 0.3%	2.0	0.1375 ± 0.9%
10	0.2618 ± 0.2%	0.2727 ± 0.5%	4.1	0.2750 ± 0.9%
20	0.4989 ± 0.1%	0.5404 ± 0.6%	8.3	0.5501 ± 0.9%
50	1.1410 ± 0.1%	1.3865 ± 1.4%	21.5	1.3753 ± 0.9%
80	1.6461 ± 0.1%	2.1727 ± 2.1%	31.9	2.2000 ± 0.9%

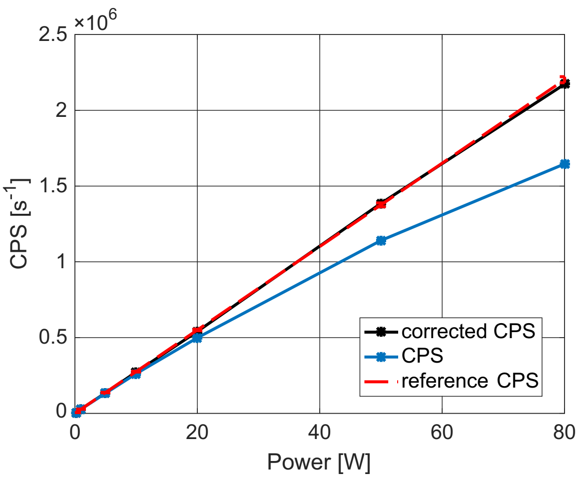

Fig. 7. Reference CPS (linear fit) and corrected CPS using the BEX method (error bars are smaller than the markers' size in the plot).

Table 3
Corrected CPS using the BEX method with a paralyzing dead time and an exponential fit.

	Corrected CPS [×10^6 s^{-1}]		
Power [W]	Polynomial fit	Exponential fit	reference CPS
0.2	0.0055 ± 0.0%	0.0055 ± 0.1%	–
1	0.0277 ± 0.1%	0.0270 ± 0.1%	0.0275
5	0.1350 ± 0.3%	0.1350 ± 0.3%	0.1375
10	0.2727 ± 0.5%	0.2727 ± 0.6%	0.2750
20	0.5404 ± 0.6%	0.5407 ± 0.8%	0.5501
50	1.3865 ± 1.4%	1.4000 ± 1.5%	1.3753
80	2.1727 ± 2.1%	2.2796 ± 2.2%	2.2000

fit models, and a maximal 2.1% difference between the results of the exponential fit model and the reference CPS. When the goodness of fit is compared, the 4th degree polynomial exhibits favorable results, as the relative value of χ increases to 0.6%.

5. Dead time correction factor - The *N* over *M* ratio

One of the basic notations in the theory of neutron dead time correction is the ratio between the theoretical count rate (*N*) and the actual count rate (*M*), referred to as the *N over M ratio* (or the *dead time correction factor*).

The two most basic models for dead time corrections are $M = \frac{N}{1+N\tau}$ for a non paralyzing dead time and $M = Ne^{-\tau N}$ in a paralyzing dead time [9]. When τ is sufficiently small (i.e., $\tau N << 1$), both models reduce to $\frac{N}{M} \approx 1 + N\tau$ In other words, in case a proper dead time correction is applied and N is properly approximated (recall, only M is measured), then a linear relation should exist between $\frac{N}{M}$ and N, passing through $\frac{N}{M} = 1$ at $N = 0$ with a slope equal to the dead time τ. Fig. 8 shows the N over M ratio obtained by implementing the BEX method to all seven experiments.

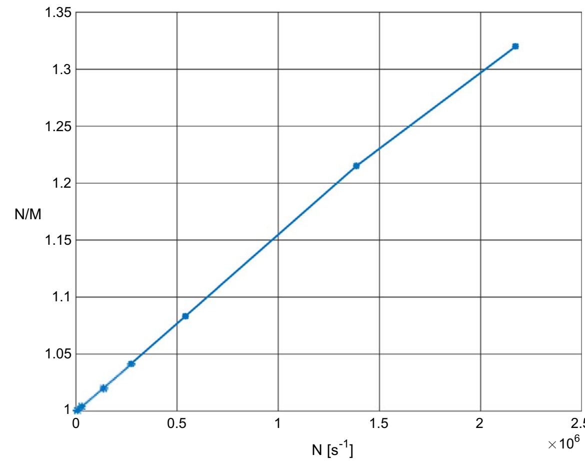

Fig. 8. The *N* over *M* ratio obtained by the BEX method.

The aim of this section is to quantify the performance of the BEX method using the *N* over *M* ratio. Using this ratio has two advantages over the linear fit with the power levels: first, the *y*-axis is dimensionless, offering a scale free quantification. Second, the results are independent of the power calibration (which by itself is subjected to uncertainties).

A very good linear fit is demonstrated in Fig. 8, with a x^2 score of x^2=1.5 ×10^{-5}. Fitting the data on a general linear mode $A + BN$ leads to A =1.001 ±0.6% and B =148.9 ±3.0% ns, showing a biasing of less than 1% from the expected values.

6. Uncertainty analysis and error propagation

Uncertainty analysis in general is done in two steps. In the first step, the errors in the different observables are quantified, and in the second, the errors are propagated onto the evaluated quantity.

The outline of the present section is to address both steps in term of the BEX method: in Section 6.1 the accuracy of the method itself is described, whereas in Section 6.2 the error propagation is discussed along with its effects on the performance of the method in terms of facility operation and physical in-pile experiments.

6.1. Uncertainty quantification on the corrected CPS

Uncertainty analysis, in general, refers to three major types of uncertainty factors: Systematic uncertainty (e.g., due to model assumptions), numerical uncertainty (e.g., due to numeric procedures applied), and statistical uncertainty (e.g., due to sampled values that might have statistical variance).

Starting with the systematic error, two main assumptions are made: First, that $f(\tau)$ can be described as a fourth degree polynomial, and second, a paralyzing dead time. Since a fourth degree polynomial is a very general model (especially for a monotonic function), the uncertainty due to the first assumption is negligible. Next, following the characteristics of the dead time in each component of the detection system (as described in Section 4.1), it is safe to state that the dead time is governed by a paralyzing model. Thus, the systematic error is estimated to be fairly small, and the uncertainty is dominated by the second and third factors described.

The numerical uncertainty is determined by two factors: uncertainty due to discretization of the induced dead time (or the step between two consecutive values of the induced dead time), and the fit

Table 4
Uncertainties in the dead time correction.

Power [W]	0.2	1	5	10	20	50	80
χ^2	2.5×10^{-3}	2.0×10^{-1}	1.9×10^{1}	1.9×10^{2}	3.7×10^{4}	6.5×10^{5}	2.8×10^{7}
statistical uncertainty[a] [%]	0.0009	0.0016	0.0033	0.0053	0.0122	0.0707	0.3260
numerical uncertainty[b] [%]	0.01	0.1	0.3	0.5	0.6	1.3	1.8

[a] $\frac{\chi}{CPS}\times 100$
[b] corresponding to discretization resolution $\Delta\tau$.

process. The resolution of imposed dead time values ($\Delta\tau$) has two effects: First, in determining the “cutoff” point from which the extrapolation should be carried out. Second, too fine resolution might results in step-wise form of the CPS, i.e., it remains constant over two or more consecutive dead time values. In the present study, this was prevented by making sure that ($\Delta\tau$) is large enough. Both effects can be translated to a “horizontal shift” of the BEX interpolation function. This, in turn, creates a vertical shift on the bisection with the y-axis, which defines the error bar on the corrected CPS.

As to the fit process, the uncertainty can be estimated by the relative value of χ (with respect to the CPS). The statistical uncertainty is determined by the statistical uncertainty on the CPS (since all the points in the BEX curve are obtained by the exact same measurement). The uncertainty on the CPS can be estimated as the square root of the CPS divided by the square root of the measurement time, and is significantly smaller than than the other two factors.

The first two uncertainty factors are summarized in Table 4.

It is obvious from Table 4 that the uncertainty is dominated by the uncertainty related to the discretiza- tion of τ. To finalize the uncertainty quantification, recall that the uncertainty on the reference values was estimated at about 1%. Therefore, the corrected CPS should all be in a 3% biasing from the reference value, which is indeed the case. To summarize, under the present parameters, the BEX method is accurate up to 2%.

6.2. Error propagation

Once the uncertainty on the corrected CPS is determined, we turn to analyze the error propagation onto the output of the experiment. Of course, this analysis can not be global, and the result is highly dependent on the actual type of experiment. Here, to demonstrate the applicability of the BEX method to actual facility operation, the error propagation is analyzed in an approach to criticalty experiment.

In the basic approach to criticality experiment, when the rector is still subcritical, the CPS (here denoted by N) and the reactivity (in natural units) are related by

$$N \approx \frac{S\times P_d}{-\rho} \tag{4}$$

where S is the external source intensity and P_d is the detection efficiency of the the monitoring system (the last is valid when the system is close to criticality and $\rho\approx k-1$). Approach to criticality is achieved by extrapolating $1/N$ to zero. In other words, the relation in Eq. (4) is used. For a general relation of the form $\rho= f(N)$, the propagation of an uncertainty in N to the uncertainty in ρ can be estimated by the simple relation:

$$\Delta\rho \approx \frac{df}{dN}\Delta N \tag{5}$$

Explicitly, in the present context, $\Delta\rho = S\times P_d \frac{1}{N^2}\Delta N$. In a typical ZPR, $S\times P_d$ is in the order of unity, and N is in the order of 10^5, which is the factor transforming from natural units of ρ to pcm. Thus, if the relative error in about 2% (meaning that $\frac{\Delta N}{N}=0.02$), the propagation to $\Delta\rho$ is in the order of 10 pcm. Of course, this is a very rough estimation, and the propagation may be higher or lower. Still, as a general note, it seems that the BEX method is suitable for dead time corrections in an approach to criticality experiment in a typical ZPR.

7. Conclusions

In this study, the BEX method for dead time corrections in a neutron detection system was introduced and studied. The method was implemented on a set of seven measurements, differing only in power level, at the MINERVE zero power reactor.

The method was implemented using two different fit models - a fourth degree polynomial and an expo- nential fit. While the results of both fitting models showed good correspondence with the reference values (obtained by a linear fit), the results of the polynomial fit were slightly better, with an average error of 0.7% and a maximal error of 1.8%. These results are consistent with the estimated uncertainty of the method (in the present setting) of about 2%.

The differences between imposing paralyzing and non paralyzing dead time within the BEX method were discussed and explained, and it was shown that for the 80 W measurement the discrepancy amounts to ~5%.

The N over M ratio was studied, once again showing favorable results, with an estimated dead time of ns, compared to the estimated dead time using the BEX plot of 150 ns.

Finally, while the data set was rather limited and coming from only one experimental setting, we conclude that the robust and high-quality performances of the method constitute sufficient proof of feasibility for the method. Yet, the method should be further validated, using a larger data set, covering a larger range of parameters (such as the power, count rate, reactor types etc.) before operational implementation.

References

[1] Jörg W. Müller, Dead-time problems, Nucl. Instrum. Methods Phys. Res. Sect. A 112 (1) (1973) 47–57.
[2] Jörg W. Müller, Generalized dead times, Nucl. Instrum. Methods Phys. Res. Sect. A 301 (3) (1991) 543–551.
[3] N. Pacilio, Bernoulli trials and counting correlations in nuclear particles detection, Nucl. Instrum. Methods Phys. Res. Sect. A 42 (2) (1966) 241–244.
[4] W. Hage, D.M. Cifarelli, Correlation analysis with neutron count distribution for a paralyzing dead-time counter for the assay of spontaneous fissioning material, Nucl. Sci. Eng. 112 (2) (1992) 136–158.
[5] Lénárd Pál, Imre Pázsit, On some problems in the counting statistics of nuclear particles: investigation of the dead time problems, Nucl. Instrum. Methods Phys. Res. Sect. A 693 (2012) 26–50.
[6] C. Dubi, I. Israelashvili, T. Ridnik, Analytic model for dead time effect in neutron multiplicity counting, Nucl. Sci. Eng. 176 (3) (2014) 350–359.
[7] F. Carloni, A. Corberi, M. Marseguerra, C.M. Porceddu, Different methods of dead-time correction in nuclear particles counting distribution, Nucl. Instrum. Methods Phys. Res. Sect. A 75 (1) (1969) 155–158.
[8] C. Michotte, M. Nonis, Experimental comparison of different dead-time correction techniques in single-channel counting experiments, Nucl. Instrum. Methods Phys. Res. Sect. A 608 (1) (2009) 163–168.
[9] Glenn Frederick Knoll, Radiation Detection and Measurement, 3rd edition, John Wiley and Sons, Inc, Orlando, 2000.
[10] T. Ridnik, C. Dubi, I. Israelashvili, J. Bagi, J. Huszti, List-mode applications for neutron multiplicity counting, Nucl. Instrum. Methods Phys. Res. Sect. A 735 (2014) 53–59.
[11] Gilles Bignan, Philippe Fougeras, Patrick Blaise, Jean-Pascal Hudelot, Frederic Mellier, (chapter 18)Handbook of Nuclear Engineering volume III, Springer, New York, NY, USA, 2010, pp. 2084–2096.
[12] P. Leconte, B. Geslot, A. Gruel, A. Pepino, M. Derriennic, J. Di-Salvo, M. Antony, R.

Eschbach, S. Cathalau, MAE- STRO: An ambitious experimental programme for the improvement of nuclear data of structural, detection, moderating and absorbing materials - first results for nat V, 55Mn, 59Co and 103Rh. In 3rd ANIMMA, 23-27 June 2013, Marseille, France, 2013.
[13] S. Douglas Nolen, The Chain-Length Distribution in Subcritical Systems. (Ph.D. thesis), Los Alamos National Lab., NM (US), June. LA-13721-T, 2000.
[14] P.´azsit Imre, Pa´l. L´en´ard, Neutron Fluctuations, Elsevier, Oxford, UK, 2008.
[15] Danielle K. Hauck, Stephen Croft, Louise G. Evans, Andrea Favalli, Peter A. Santi, Jonathan Dowell, Study of a theoretical model for the measured gate moments resulting from correlated detection events and an extending dead time, Nucl. Instrum. Methods Phys. Res. Sect. A 719 (2013) 57–69.
[16] B. Geslot, F. Berhouet, L. Oriol, S. Braud, C. Jammes, P. Filliatre, J.F. Villard, Development and manufacturing of special fission chambers for in-core measurement requirements in nuclear reactors. in: Proceedings of the First International Conference on Advancements in Nuclear Instrumentation Measurement Methods and their Applications (ANIMMA), pages 1–pages 4, June, 2009.