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Editorial

Special issue: Advances in pattern formation

Pattern formation refers to the emergence of regular (albeit often complex) spatiotemporal variations of certain observables in systems that are driven away from thermodynamic equilibrium. It is a generic behavior of spatially-extended nonlinear systems and crosses a wide variety of natural settings and laboratory experiments. Examples include stationary periodic patterns of pigments on animal skins and of vegetation patches in semi-arid climates, spiral waves in auto-catalytic chemical reactions and in intracellular calcium activity, spatially localized states (e.g., solitons in optics, action potentials in physiology, and oscillons in Faraday wave-type systems), domain walls of magnetization in ferromagnets, buckling and wrinkling of thin sheets, charge-induced self-assembly in electrolytes and polymer mixtures, localized resonances in the auditory system, dynamics in networks, animal and insect swarming, branching in material science and biology, and actin filament polymerization in biological cells that triggers the development of intracellular protrusions and cell motility.

The intense effort to understand pattern-formation mechanisms and pattern dynamics during the late 20th century, has gradually given place to studies of specific natural contexts related to both animate and inanimate matter. Along with this drift of interests, new questions of pattern formation have emerged especially in the context of biological and ecological contexts, which can be classified as questions related to (i) the rich spatio-temporal behaviors that have been predicted by mathematical models, (ii) the functioning of living systems in variable environments, and (iii) the inevitable need to intervene with their dynamics without impairing their function. Uncovering mechanisms that drive pattern-formation phenomena in inanimate and animate matter is of equal importance also for developing technological applications and for understanding functional aspects of living systems. On the other hand, studies of pattern-forming systems are also fertile sources of new mathematical questions that advance the development of analytical and numerical methods, which, in turn, contribute new insights into the original questions.

This special issue (SI) emerged out of an international workshop titled "*Advances in pattern formation: New questions motivated by applications*". The workshop aimed to address cutting-edge advances in a wide range of research fields where pattern formation phenomena play or are expected to play essential roles, focusing on common threads that go across different research areas, encouraging cross-fertilization, and highlighting new outstanding open questions. The workshop was also an occasion to celebrate the 65th birth year of Ehud Meron, acknowledging his pioneering contributions to pattern formation in general and vegetation patterns in particular.

The event took place, on February 18–21 2019, at the Blaustein Institutes for Desert Research at the Sede Boqer campus of Ben-Gurion University of the Negev, known for its spectacular and inspiring desert landscape. The workshop assembled about 80 scientists that work on distinct aspects of pattern formation, both senior and early-carrier researches, see group-photo in Fig. 1; some participants cordially contributed their original studies to this SI. We characterize the contributions as belonging to three categories according to their motivation, origin and/or applications: Inanimate media [1–6], biological systems [7–11], and ecological pattern formation [12–17].

1. Briefly about Ehud Meron

Ehud Meron is a Phyllis and Kurt Kilstock Chair Professor in Environmental Physics of Arid Zones at the Blaustein Institutes for Desert Research, Ben-Gurion University of the Negev. His research interests are nonlinear physics, theoretical ecology and complex systems. Ehud has made important contributions to the understanding of front dynamics in activator–inhibitor systems and in periodically forced oscillatory systems, highlighting the roles that front instabilities play in initiating and nurturing complex spatio-temporal patterns. In his studies he has collaborated with leading experimentalists (Harry L. Swinney and others) on testing theoretical predictions mostly in experiments on continuously fed unstirred chemical reactors. His more recent research focuses on water-limited ecosystems, where he is known for his pioneering contributions to the understanding of vegetation pattern formation and its implications to desertification, biodiversity and ecosystem sustainability. He is the author of a monograph entitled "Nonlinear Physics of Ecosystems" [18] that aims to bridge the gap between the disparate, and yet closely related, research fields of pattern formation and spatial ecology.

2. About the contributions

We start with contributions on inanimate matter: Ariel and Schiff [1] explain using analytical and numerical methods, why Lévy walks suggest super-diffusion for self-propelled particles, leading to an apparently new route for Lévy walking phenomenon

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Fig. 1. A group photo taken at the end of the workshop, capturing the majority of the participants. Ehud Meron is sitting at the bottom row and in the middle of the group of three people who raise their hands, where on the left is Ehud's spouse Liora.

in chaotic systems. Avalos et al. [2] aim to advance a framework that can bridge between mesoscopic properties to macroscopic performance in material science. The authors use computed tomography (CT) images and phase field modeling of fracture toughness to demonstrate consistency with experimental results of epoxy resins. By considering coupled Brusselator (reaction–diffusion) systems, Castelino et al. [3] study the role of three-wave interactions. They demonstrate the conditions when steady patterns appear, such as quasipatterns, along with the emergence of spatiotemporal chaos along with applicability of the results to Faraday wave patterns. Gavish [4] analyzes the Poisson–Nernst–Planck–Lennard-Jones (PNP–LJ) model in the context of highly concentrated electrolytes. The author reveals that when going beyond the leading order approximation of the LJ interaction kernel, a distinct class of steric PNP equations emerge, a consequence that is related to pattern formation in electrically charges solutions. Ly et al. [5] study deposition patterns using continuum models, a Cahn–Hilliard model for Langmuir–Blodgett transfer and a thin-film equation for acoustic wave-driven coating. The authors reveal the bifurcation diagram for 2D steady states that correspond to deposition patterns. Uecker and Wetzel [6] demonstrate the advances of the numerical continuation method employing the bifurcation package *pde2path*. Analyzing the Brusselator model in 3D, they compute the homoclinic snaking branches of planar fronts between body centered cubes (BCCs) and the spatial homogeneous solution, planar fronts between BCCs and tubes, and moving fronts between lamellas and tubes. At the end, the authors suggest that such patterns can be observed in microemulsions.

Biological contributions include the following studies: Champneys et al. [7] present a comparative study focused on the formation mechanisms of localized patterns and fronts in reaction–diffusion equations on long 1D domains. Using the semistrong limit analysis, they show several applications to natural systems, such as cellular polarity formation and the transitions between vegetation states on continental scales. Inspired by localized oscillations, believed to play an important role in frequency discrimination of incoming sound waves, Edri et al. [8] employ the forced complex Ginzburg–Landau (FCGL) equation in a spatially nonuniform 1D domain, subjected to both parametric and additive forcing, to study the spatial profile of the oscillations and the factors that affect it. Motivated by shape changes of eukaryotic cells during motility, Moreno et al. [9] study several motility scenarios using a combined reaction–diffusion and phase-field system. Employing numerical simulations and live cell imaging experiments, the authors identify the key parameters of the mathematical model that determine the different motility regimes. Shulman et al. [10] introduce a model-independent control algorithm to address problems in which a complex network needs to be altered to a pre-specified target state, such as how to alter a disease-induced tissue to a healthy state. Using nonlinearly-coupled electrical circuits to pre-specified target states, the authors show that a feedback based on 'response manifolds', is needed to design the necessary control. Vo et al. [11] demonstrate multi-mode attractors (MMAs) in reaction–diffusion systems with slow–fast kinetics that may arise in neural and cardiac models. The authors relate these MMAs to different regions of the spatial domain exhibiting different modes of (temporal) oscillation, such as spiking modes, bursting modes and alternating modes. In particular, they show that MMAs exhibit new types of maximal spatio-temporal canards that lie in the transition intervals between adjacent regions in which the MMA exhibits different oscillatory modes.

Finally, we conclude with contributions inspired by ecological patterns: Motivated by multi-variable cyclic ecological systems, Bayliss et al. [12] extend the rock–paper–scissors approach. Using a May–Leonard type model, they analyze four- and five-species systems and show different behavior and consequences related to heteroclinic connections, while also elucidating possible extensions to systems of higher order. Motivated by ecohydrological processes, Gandhi et al. [13] propose a novel reaction–advection–diffusion based switching model with processes describing slow and fast timescales. Through analysis the authors show several agreements with observations, such as band spacing and upslope colonization rates, and formulate a framework for investigating the possible impact of changes to frequency and intensity of rain events in dryland ecosystems. Jaibi et al. [14] use singular perturbation theory (i.e., a slow–fast setting) to establish the existence of a multitude of heteroclinic/homoclinic/periodic orbits in a two-component reduction of the Gilad et al. model for dryland ecosystem dynamics. Specifically, the authors construct novel multi-front solutions and discuss their generic properties. Motivated by clonal plant growth, Ruiz-Reynés et al. [15] formulate a model with local and nonlocal interactions, and study the observations of spatial structures in Posidonia oceanica meadows in the Mediterranean Sea. Specifically, they focus on spatially extended and localized structures. For the former a transition between pushed and pulled fronts is identified, while for the latter distinct snaking bifurcations diagrams are computed. Siero [16] studies three vegetation models (proposed by Klausmeier, Rietkerk et al. and Gilad et al.) to identify similarities and differences that are related to the impacts of soil and surface water components, on the Turing-type instability. Tlidi et al. [17] perform a reduction of an integro-differential model for vegetation to a partial-differential system and study periodic and localized patterns. They find that in this reduced model patches always exhibit repulsive interactions whereas gaps can form grouped solutions under appropriate conditions.

3. Concluding remarks

Evidently, the contributions in this SI are merely the tip of an iceberg for recent developments of analytical and numerical methods that allow tackling nonlinear behaviors. They also demonstrate the importance of a multi-disciplinary approach to real-world applications involving patterns. We end this prolog to the SI by wishing Ehud Meron good health and hope that his works will continue to inspire the scientific community for many years to come. We also want to thank all the contributors for sharing their studies, express our gratitude to V. M. Pérez García, the Deputy Editor of Physica D, for both inviting and assisting through completion of this SI, and finally, convey our appreciation to S. Natarajan for all the administrative coordination.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Arik Yochelis *
Department of Solar Energy and Environmental Physics, Blaustein Institutes for Desert Research, Ben-Gurion University of the Negev, Sede Boqer Campus, 8499000 Midreshet Ben-Gurion, Israel
E-mail address: yochelis@bgu.ac.il.

Erez Gilad
The Unit of Nuclear Engineering, Ben-Gurion University of the Negev, Beer Sheva 8410501, Israel

Yasumasa Nishiura
Research Institute for Electronic Science (RIES), Hokkaido University, Sapporo 060-0812, Japan

Mary Silber
Department of Statistics and Committee on Computational and Applied Mathematics, University of Chicago, Chicago, IL 60637, USA

Hannes Uecker
Institute for Mathematics, Carl von Ossietzky University of Oldenburg, 26111 Oldenburg, Germany

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* Corresponding editor.