Linear Stability Analysis of HTR-like Micro-reactors

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INTRODUCTION

The technology of nuclear micro-reactors offers many promising advantages and benefits. Their small size allows their deployment in remote locations. The simple design of such reactors requires fewer components and facilitates their maintenance. Fast installation can allow micro-reactors to be connected and generate power shortly after arriving on site.

Several micro-reactors concepts are currently being designed. A class of these designs are based on a small-scale high-temperature gas-cooled reactor (HTR), utilizing TRISO fuel particles with a graphite moderator. This work focuses on the stability of these HTR-like micro-reactors.

Reactivity feedback mechanisms play an important role in the safe operation of nuclear reactors in general, and in self-regulation and passive safety in particular. Such mechanisms are quantified by reactivity coefficients. Usually, safety guidelines require all reactivity coefficients to be negative, in order to ensure safe operation. However, it has been demonstrated in the past that the dynamics of a reactor core may become unstable to small perturbations even when all reactivity coefficients are negative [1].

Feedback mechanisms are usually nonlinear. There exist several approaches to study the stability of nonlinear systems to small perturbations around a steady-state. According to the first method of Lyapunov, the stability of a system is analyzed using the eigenvalues of the Jacobian matrix of the system. When real parts of all eigenvalues are negative, the steady-state is asymptotically stable and the system is stable.

An equivalent approach is Laplace-domain (also known as frequency-domain) analysis. In this approach, the Laplace transform is used to derive the transfer function of the system, representing its response to small perturbations. The linear stability is analyzed based on the poles of this function, which are identical to the eigenvalues of the Jacobian [2].

Due to the complexity of reactor systems, stability studies of such systems usually rely heavily on numerical approaches, and therefore only a small fraction of the possible reactor states can be practically analyzed. In order to overcome this inherent limitation, reduced-order reactor models are developed and studies, such that more general insights can be obtained, sometimes even analytically. In previous studies, a Two-Temperature Reactor Model (TTRM) was utilized, comprising of the point kinetics equations (PKE) coupled to heat balance equations of fuel and coolant temperatures [3, 4]. Several variations of this model have been proposed. Some studies have derived linear stability criteria using a graphical method (Nyquist Plot), while others have derived them via the exact Routh-Hurwitz (RH) conditions.

In a recent study [5], full analytic stability criteria for a 4-equation TTRM were derived - by analyzing the poles of the transfer function using the RH conditions. The study has shown that even when both reactivity coefficients are negative, the reactor may become linearly unstable. Moreover, it was shown that depending on the reactor parameters, this instability may be either power dependent or independent, i.e. exist only for power levels exceeding a certain threshold or for all power levels, respectively.

In this study, linear stability analysis is performed in order to investigate the influence of reactivity feedback mechanisms on the behavior of HTR-like micro-reactors, using the state-space representation approach. The Holos-Quad concept, which was recently proposed by HolosGen, LLC, is used as an HTR-like micro-reactor benchmark [6], since this design has a relatively complete amount of information in the public domain, and the analysis is based on a TTRM of this design. The parameter values are carefully chosen to fit the special characteristics of the Holos-Quad micro-reactor.

It should be emphasized that the stability in this study is analyzed under the assumption that no reactivity control system is present, to assess the inherent safety of the design. When such a system is present, it can stabilize the dynamics by responding to the reactivity perturbations and decreasing them.

HOLOS-QUAD MICRO-REACTOR

A new micro-reactor concept has been recently proposed by HolosGen, LLC [7]. It is based on an advanced high-temperature helium-cooled reactor, fueled by TRISO particles distributed in a graphite block. With a nominal (thermal) power of 22MW, the Holos-Quad concept consists of four Sub-critical Power Modules (SPMs) that fit into one 40-foot ISO container.

The Holos design currently considers several options, both for primary and secondary control systems. For the primary system, the control consists of the capability to mange reactivity by moving SPMs apart, thus increasing neutron leakage. The secondary system includes a neutron absorbing blade inserted in between the four SPMs during the transport of the reactor, and rotating control drums during its operation.

In this study, two reactivity feedback mechanisms of the

Holos-Quad are considered, namely moderator density and fuel temperature. The effect of helium temperature on neutronic behavior is negligible. Other feedback mechanisms may be considered in future studies, such as axial and radial expansions of the fuel [8].

A detailed neutronic design and analysis of the Holos-Quad concept was performed by Argonne National Laboratory [6]. In this analysis, which includes full-core simulations using both Monte Carlo and the deterministic code PROTEUS, reactivity feedback constraints were evaluated. The Moderator Density Coefficients (MDC) is calculated by reducing the graphite density of the fuel pin matrix and the graphite block by 1%. The Fuel Temperature Coefficient (FTC) is calculated by increasing the temperature of the fuel by 300°K.

According to the requirements of the design, both coefficients (MDC and FTC) should be negative in order to ensure reactor safety. The results of the ANL benchmark yield a MDC of -145 pcm/%density, which is equivalent to -0.3pcm/°K of moderator temperature, taking into account the thermal expansion of the graphite, while the FTC is almost an order of magnitude bigger, with -3.2pcm/K. The reason for the small magnitude of the MDC is due to the presence of diluted burnable poison in the graphite moderator. Hence, the primary negative neutronic feedback in the benchmark is the fuel temperature reactivity feedback.

REDUCED-ORDER MODEL FOR HOLOS

The TTRM model for the Holos Reactor includes both neutronic and thermal-hydraulic (heat balance) equations. The neutronic part of the model is based on the PKE, assuming a single delayed group for further simplification yields:

$$\frac{dP(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda}P(t) + \lambda c(t), \quad (1)$$

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda}P(t) - \lambda c(t), \quad (2)$$

where P is the reactor power, ρ is the reactivity, β is the effective delayed neutron fraction, Λ is the neutron generation time, λ is the decay constant of the delayed precursors and c is the (effective) delayed precursor concentration. The delayed fraction and decay constant are obtained from the 6-group data using $\beta = \sum_i \beta_i$ and $\lambda = \sum_i \beta_i \lambda_i / \beta$, where β_i and λ_i are the group delayed neutron fraction and decay constant, respectively.

In order to derive the heat-balance equations, the 2D model of the coolant channel is analyzed. In the Holos benchmark, this model consists of five layers: fuel, graphite, lead-buffer, clad and coolant [6], each layer having its own temperature. In order to simplify the model, only the fuel and moderator layers are considered for the heat-balance equations. This is motivated by the fact that only these two materials have reactivity feedback, while the other three do not significantly affect the neutron dynamics.

To derive the fuel temperature equation, we assume that a certain fraction of the generated fission power is deposited directly in the graphite matrix, due to gamma heating and slowing down of neutrons. This effect is represented by the dimensionless parameter q_f, which represents the fraction of the generated power deposited in the fuel. The heat balance equation for the fuel temperature is

$$m_f c_f \frac{dT_f(t)}{dt} = q_f \cdot P(t) - hA\left(T_f(t) - T_m(t)\right), \quad (3)$$

where m_f and c_f are fuel mass and specific heat capacity, respectively, and T_f and T_m are fuel and graphite mean temperatures, respectively. The parameters h and A are the heat transfer coefficient and the surface area between the fuel and moderator, respectively.

The graphite moderator is cooled by the helium flow in the cooling channels. Considering heat removal from the moderator, several modeling approaches can be employed. Here, we assume a constant heat removal rate from the moderator by the coolant. A different approach, which takes into account the flow rate of the coolant, assumes that inlet coolant temperature is constant. This approach may be studied in the future. The heat balance equation for the moderator is

$$m_m c_m \frac{dT_m(t)}{dt} = (1-q_f)\cdot P(t) + hA\left(T_f(t) - T_m(t)\right) - P_r, \quad (4)$$

where m_m and c_m is the moderator mass specific heat capacity, respectively, and P_r is the constant heat removal rate. The assumption of constant heat removal is valid in the vicinity of steady-state, such that the parameter P_r is equal to the steady-state power.

Finally, the reactivity model includes reactivity coefficients of both fuel and moderator temperatures:

$$\rho(t) = \rho_0 + \rho_{fb}(t) = \rho_0 + \alpha_f \Delta T_f(t) + \alpha_m \Delta T_m(t), \quad (5)$$

where ΔT_f and ΔT_m represent deviations from steady-state temperatures.

In order to estimate the kinetic parameters, we used the Modular High-Temperature Gas-Cooled Reactor (MHTGR) benchmark published recently by NEA [9]. Although it is not a small-scale reactor, it utilizes similar types of fuel, moderator and coolant. The kinetic parameters are given in Table I. Typical values of specific heat capacity were also estimated using this report. The specific heat capacity of the fuel was calculated by mass-averaging the specific heat capacities of the different layers in the TRISO particle.

Other parameter values, such as heat transfer coefficients, are also based on HTR reactor studies. The total mass of fuel and moderator and the contact area between them, were estimated based on the geometric description of the ANL Holos benchmark [6]. A complete list of parameter values is given in Table II.

Note that since the design of the Holos micro-reactor is still evolving, and some design features are propriety, the values listed above are subjected to amendments as the design firms up. Specifically, the power P_s might change, and the values of m_f, m_m and A might change accordingly. In addition, for a given core design, some values may depend on the operational conditions, e.g. dependence of the reactivity coefficients on core temperature and/or distance between SPM units. Therefore, it is important to analyze sensitivity of the results to deviations in these values. In this study, sensitivity to reactivity coefficients (α_f, α_m) is demonstrated, while sensitivity to other parameters may be investigated in the future.

TABLE I: Kinetic Parameters for Holos Model

Group	β_i ($\times 10^{-4}$)	λ_i (s^{-1})
1	1.42	0.0127
2	9.24	0.0317
3	7.80	0.116
4	20.66	0.311
5	6.71	1.40
6	2.18	3.87

TABLE II: Nominal parameter values for the Holos microreactor reduced-model.

Parameter	Value	Units	Ref.
Λ	1680	μs	[10]
m_f	2002	kg	[6]
c_f	977	J/°K/kg	[9]
q_f	0.96	1	[11]
h	1999	W/m^2/s	[12]
A	4712	m^2	[6]
m_m	11573	kg	[6]
c_m	1697	J/°K/kg	[9]
P^s	22	MW	[6]
α_f	-3.2	pcm/°K	[6]
α_m	-0.3	pcm/°K	[6]

LINEAR STABILITY ANALYSIS

The steady-state solution of Eqs. (1)-(4) is obtained with zero total reactivity ($\rho = 0$) and non-zero steady-state power (P^s). The power is a free variable, such that there exists a steady-state for each value of P^s. Moreover, one of the two temperatures, T_f or T_m, is also a free variable. In steady-state, $P_r = P^s$ and $q_f P^s = hA(T_f - T_m)$.

It is convenient to rewrite the equations using the following definitions of inverse-time parameters: $\lambda_g = \beta/\Lambda, \lambda_f^\alpha = P^s\alpha_f/\Lambda hA, \lambda_m^\alpha = P^s\alpha_m/\Lambda hA, \lambda_f = hA/m_f c_f$, and $\lambda_m = hA/m_m c_m$. In addition, the variables P and c are normalized by hA, such that all variables have temperature units.

Linearizing around some steady-state solution, Eqs. (1)-(4) can be written in the following matrix representation, where each element of the linearized matrix have inverse-time units:

$$\frac{d}{dt}\begin{pmatrix}\delta P(t)/hA\\ \delta c(t)/hA\\ \delta T_f(t)\\ \delta T_m(t)\end{pmatrix} = \begin{pmatrix}-\lambda_g & \lambda & \lambda_f^\alpha & \lambda_m^\alpha\\ \lambda_g & -\lambda & 0 & 0\\ q_f\lambda_f & 0 & -\lambda_f & \lambda_f\\ (1-q_f)\lambda_m & 0 & \lambda_m & -\lambda_m\end{pmatrix}\begin{pmatrix}\delta P(t)/hA\\ \delta c(t)/hA\\ \delta T_f(t)\\ \delta T_m(t)\end{pmatrix}. \quad (6)$$

The matrix in Eq. (6) is the Jacobian matrix, also known as the (linear) system matrix. As discussed above, the linear stability is guaranteed if all eigenvalues have a negative real part.

It is important to point out that linear stability analysis only describes the dynamics in the vicinity of the fixed point, assuming small deviations from steady-state. When perturbations increase, nonlinear terms cannot be neglected anymore.

The linearized model therefore includes six inverse-time parameters ($\lambda_g, \lambda, \lambda_f^\alpha, \lambda_m^\alpha, \lambda_f, \lambda_m$) and one dimensionless parameter (q_f). Indeed, it can be seen that 12 out of 16 elements in the system matrix are non-zero, of which only 7 are independent.

Notice that the power and reactivity coefficients appear only in the parameters λ_f^α and λ_m^α, in the form of their product: $\lambda_f^\alpha \propto P^s\alpha_f$ and $\lambda_m^\alpha \propto P^s\alpha_m$.

The eigenvalues of the system matrix (6) are the zeros s_i of the characteristic polynomial $det(s_i I - A) = 0$ where A is the system matrix, and I is the identity matrix. The system matrix is four-dimensional, therefore it has a fourth-order characteristic polynomial. The linear stability can be analyzed by using the RH conditions for coefficients of a fourth-order characteristic polynomial. With this, the following inequalities are obtained:

$$A = \lambda_f^\alpha + \lambda_m^\alpha < 0, \quad (7)$$

$$B = \lambda_f(\lambda_m + q_f\lambda)\lambda_f^\alpha + \lambda_m(\lambda_f + q_m\lambda)\lambda_m^\alpha < 0, \quad (8)$$

$$\lambda_f\left[q_f(\lambda_f + \lambda_g) - q_m\lambda_m\right]\lambda_f^\alpha + \lambda_m\left[q_m(\lambda_m + \lambda_g) - q_f\lambda_f\right]\lambda_m^\alpha < (\lambda_f + \lambda_m + \lambda + \lambda_g)(\lambda_f + \lambda_m)(\lambda + \lambda_g) - \lambda_f\lambda_m\lambda(\lambda_f + \lambda_m + \lambda + \lambda_g)^2\frac{A}{B}. \quad (9)$$

where $q_m = 1 - q_f$ was introduced to emphasize the symmetry $f \leftrightarrow m$. The term B/A is bounded by $\lambda_f(\lambda_m + q_f\lambda)$ and $\lambda_m(\lambda_f + q_m\lambda)$ which are independent on λ_f^α and λ_m^α. Hence, the RHS of Eq. (9) can be neglected when the LHS is much larger (in absolute value), i.e. for large values of λ_f^α and λ_m^α. With this approximation, Eq. (9) now depends linearly on λ_f^α and λ_m^α:

$$\lambda_f\left[q_f(\lambda_f + \lambda_g) - q_m\lambda_m\right]\lambda_f^\alpha + \lambda_m\left[q_m(\lambda_m + \lambda_g) - q_f\lambda_f)\right]\lambda_m^\alpha < 0. \quad (10)$$

RESULTS

A 2D stability map is drawn in Fig. 1, in the parameter space spanned by λ_f^α and λ_m^α. This map was calculated by Eqs. (7) - (9) using the values from Tables I and II. The black line separates the stable and unstable regions.

An operational region is represented by the blue area in Fig. 1. This region covers all the possible values of power and reactivity coefficients. It is spanned the straight lines with slope $\lambda_m^\alpha/\lambda_f^\alpha = \alpha_m/\alpha_f$, where α_m and α_f are possible values for Holos. A variation of ±50% was assumed for α_m and α_f with regards to their nominal value of Table II. It can be seen that the assumed operational region is always stable, i.e. stability is guaranteed for any steady-state power level (unconditional stability).

The stability dependence on reactivity coefficients can be demonstrated for specific power levels, as depicted in Fig. 2. Here, the inverse-time variables (λ_f^α, λ_m^α) were transformed back to the reactivity coefficients (α_f, α_m) assuming nominal power (22MW). At this power level, the stability boundary is well approximated by only two linear inequalities, Eq. (7) and

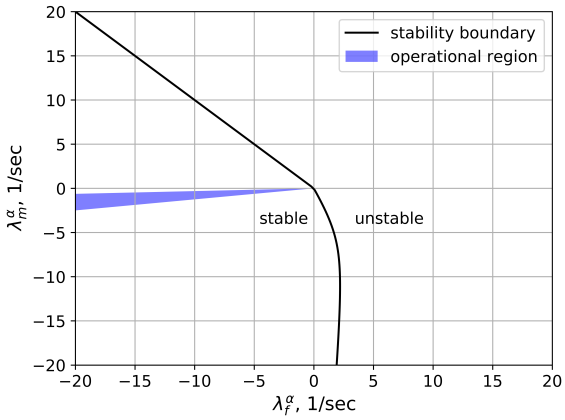

Fig. 1. Stability map for the parameter space spanned by λ_f^α and λ_m^α. The operational region is represented by the blue area.

the approximated Eq. (10). As in Fig. 1, the nominal values are shown with variation range of ±50%. As depicted in the figure, the parameter values of Holos yield a reactor that is well inside the stable region. There exists an unstable region with negative coefficients, however it requires significantly larger magnitudes of moderator temperature coefficient and significantly smaller magnitudes of fuel temperature coefficient.

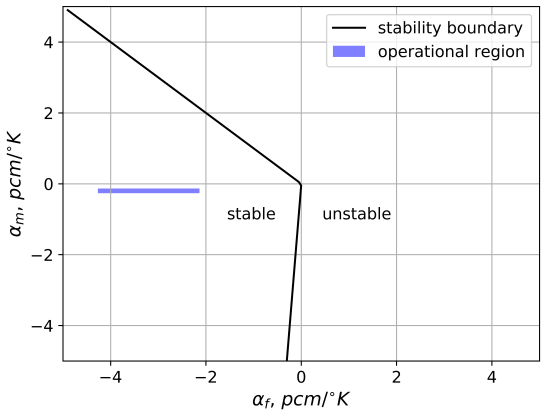

Fig. 2. Dependence of linear stability on reactivity coefficients, for reactor power of 22MW. The operational region is based on nominal values variation range of ±50%.

CONCLUSIONS

Linear stability analysis of an HTR-like micro-reactor was presented, based on the Holos-Quad benchmark. Analysis was carried using a reduced-order model, consisting of point kinetics equation coupled to heat balance equations for fuel and moderator temperatures. The values of the parameters were estimated based on both Holos ANL benchmark [6] and on other high-temperature gas-cooled reactor systems. The analysis shows the stability of the reactor to small perturbations, in the absence of control system.

The results show that, based on the nominal values of the Holos parameters, the reactor is well inside the stable region, including variation range of ±50% in the reactivity coefficients. Several parameter values may be changed in the future, depending on both reactor design and operational conditions. Sensitivity of the results to parameters variation should be studied in the future, in order to show the robustness of the linear stability of the Holos design.

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